Correlation functions in Landau-gauge QCD

RBRC Workshop

QCD in Finite Temperature and Heavy Ion Collisions

Nils Strodthoff, LBNL

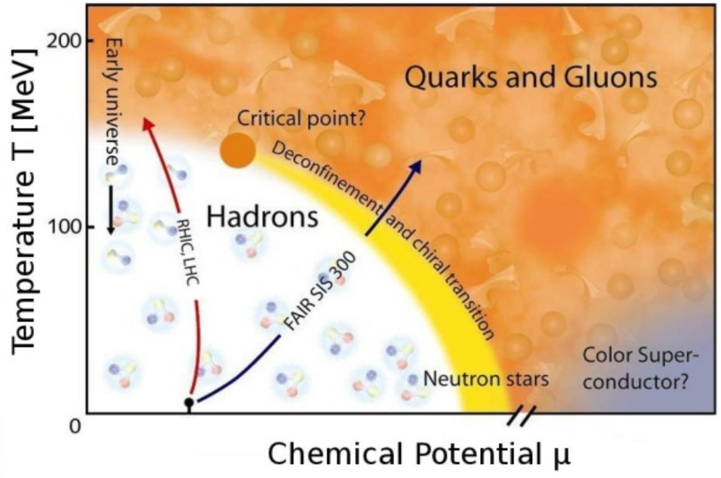
In collaboration with A. Cyrol, M. Mitter and J. Pawlowski

DFG Deutsche Forschungsgemeinschaft

BERKELEY LAB

QCD Phase Structure

Aim: Quantitative understanding of the **phase structure of QCD**



➢ adapted from GSI

Phase structure at large chemical potentials largely unknown due to **sign problem** in lattice QCD…

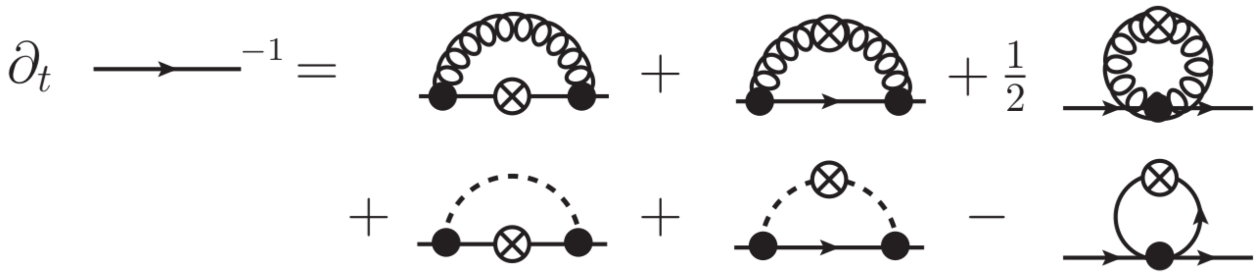
Quantitative understanding requires **first-principle approaches**

- Lattice QCD
- Functional approaches
    - ✓ Complementary to the lattice
    - ✓ No sign problem
    - ✓ Calculation of realtime observables

Functional Approaches

- Dyson-Schwinger equations (DSE)
- n-particle irreducible methods (nPI)
- **Functional Renormalization Group (FRG)**

use relations between off-shell Green's functions



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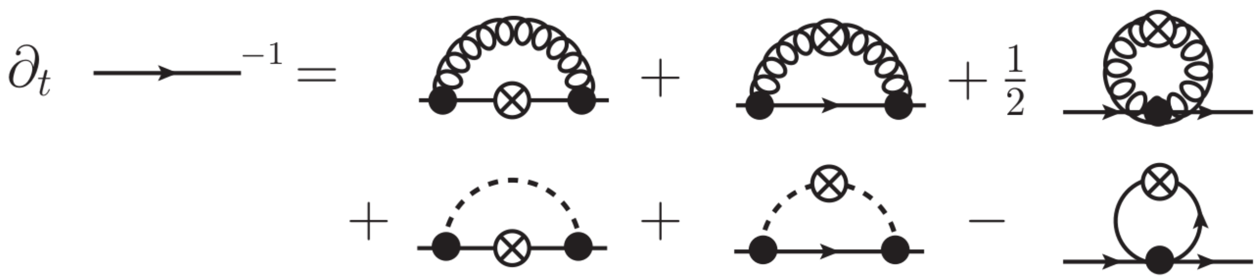


Top-down approach: **fQCD collaboration**

J. Braun, L. Corell, A. K. Cyrol, W.-J. Fu, M. Leonhardt, M. Mitter, J. M. Pawlowski, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink

Quantitative continuum approach to QCD in the FRG framework

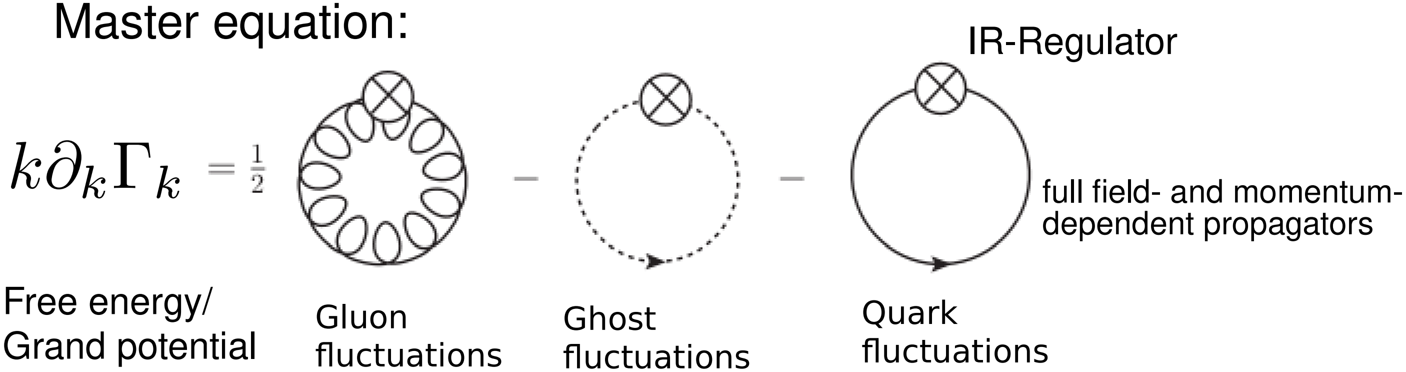
✓ No phenomenological input
✓ Input parameters: fundamental parameters of QCD

Functional RG for QCD

Spirit of **Wilson RG**: Calculate full quantum effective action by integrating fluctuations with momentum k

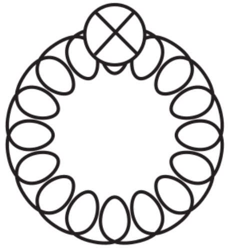
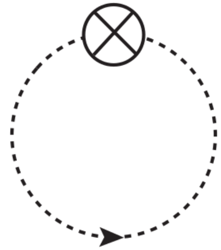
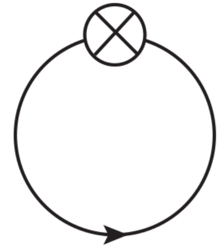
Functional Renormalization Group (FRG)

Master equation:



+flow equations for n-point functions via functional differentiation

Dynamical Hadronization

$$\frac{\mathrm{d}\Gamma_k}{\mathrm{d}t} = \frac{1}{2}$$  $-$  $-$ 

$t = \log(k/\Lambda)$

Free energy/
Grand potential

Gluon
fluctuations

Ghost
fluctuations

Quark
fluctuations

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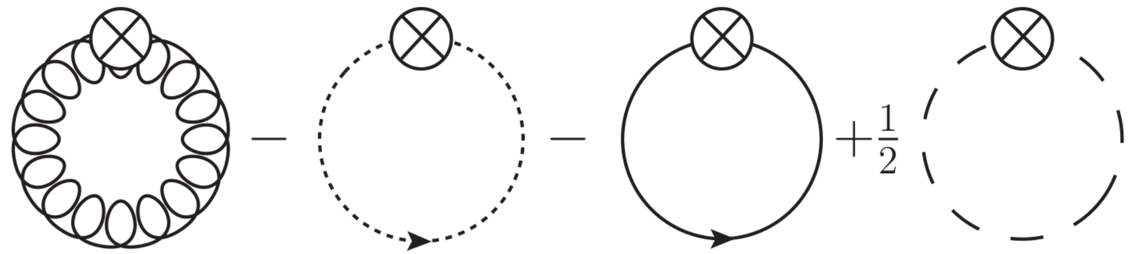


$t = \log(k/\Lambda)$

Free energy/ Grand potential

Gluon fluctuations

Ghost fluctuations

Quark fluctuations

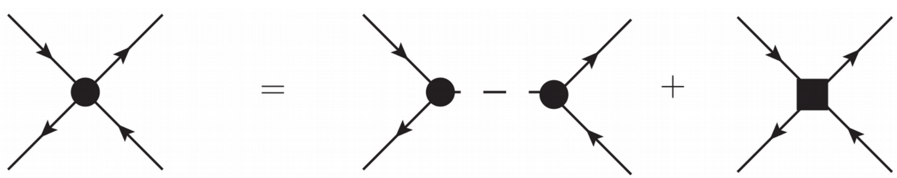
Hadronic fluctuations here: (σ,π)

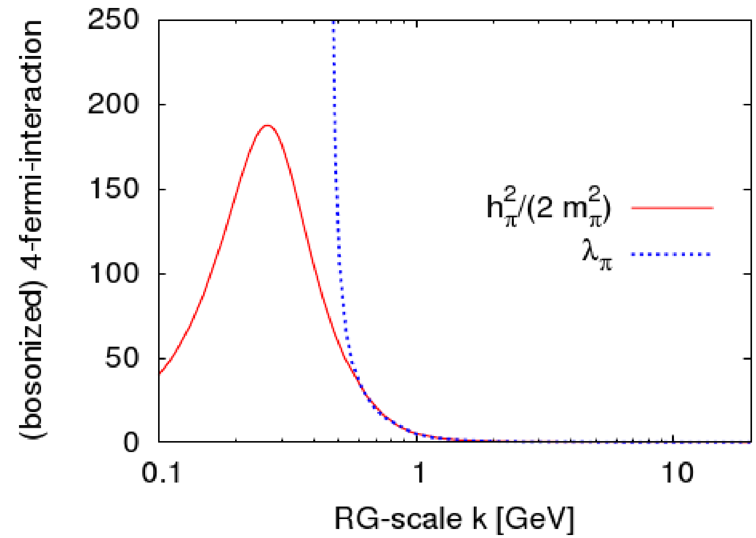
Efficient bookkeeping
no double counting

Dynamical hadronization

Store resonant 4-Fermi structures in terms of effective mesonic interactions

- Gies, Wetterich Phys.Rev. **D65** (2002) 065001





✔ Effective models incorporated

- cf. talks of J. Eser and Z. Szép

- initial conditions determined by QCD-flows

Vertex Expansion in QCD

Perturbative relevance counting no longer valid:

- Requires non-perturbative expansion schemes
- Here: **Vertex expansion** in terms of 1PI vertex functions

$$\Gamma[\Phi] = \sum_n \int_{p_1,\ldots,p_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(p_1,\ldots,p_{n-1})\Phi^1(p_1)\cdots\Phi^n(-p_1-\cdots-p_{n-1})$$

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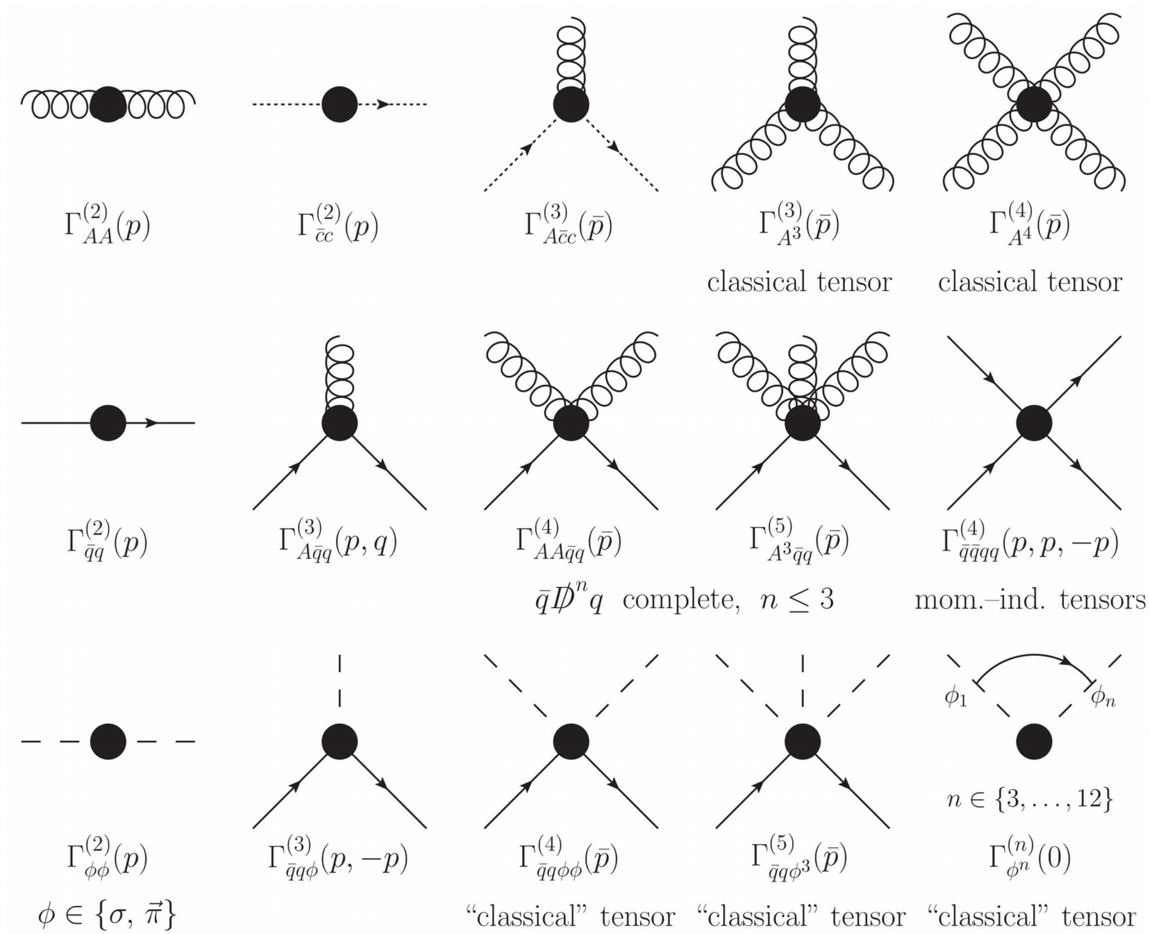
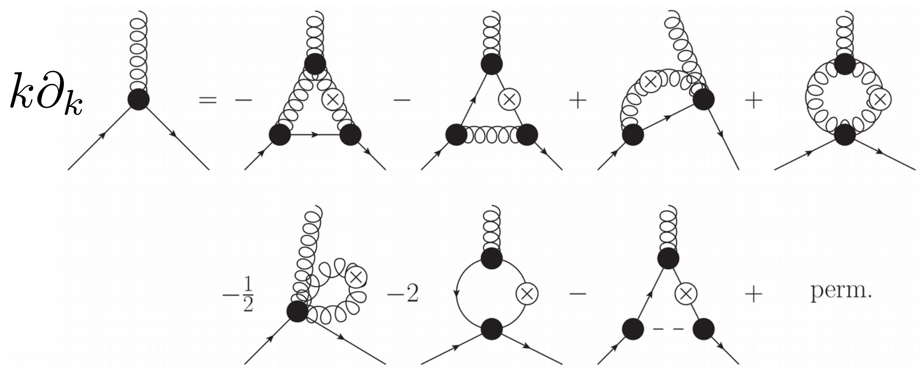


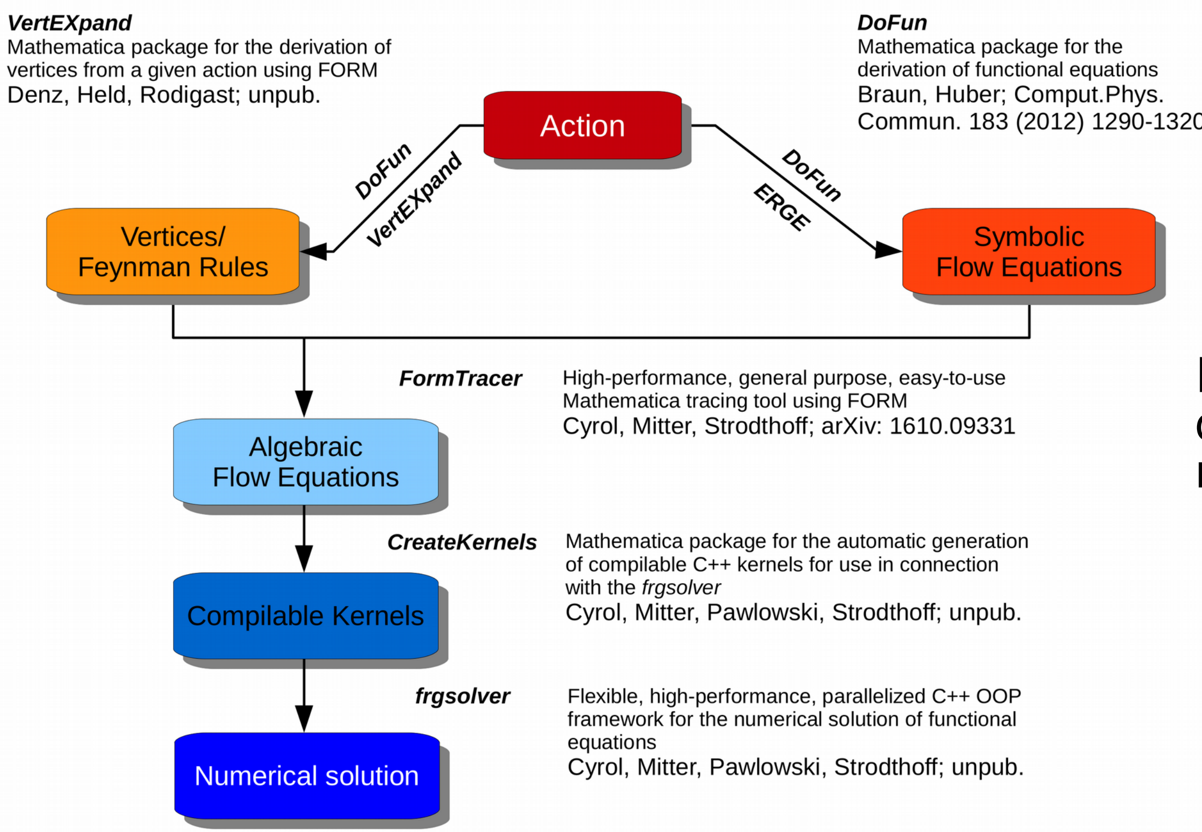
...aiming at apparent convergence.

Towards 1-Click QCD

e.g. quark-gluon vertex equation:



self-consistent solution



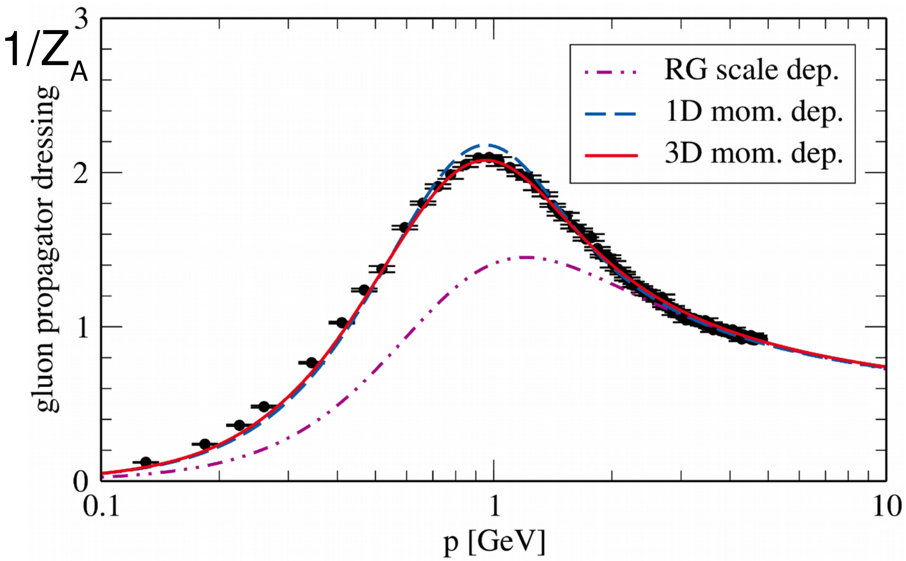
Requirement for dedicated computer-algebraic and numerical tools

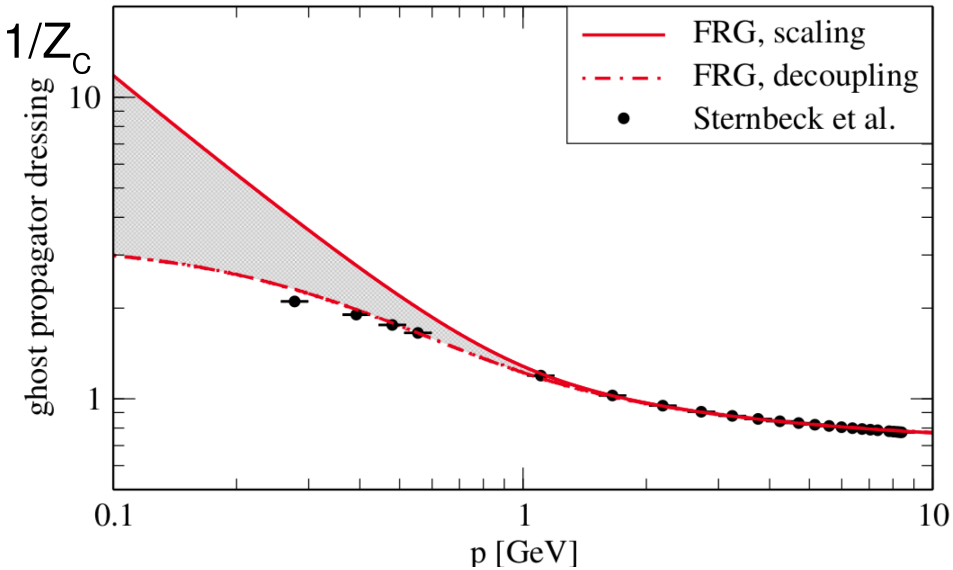
YM Propagators

Self-consistent solution of the system of transversal 2-,3- and 4-point functions

- Cyrol, Fister, Mitter, Pawlowski, NSt Phys.Rev. **D94** (2016) no.5, 054005

$$[\Gamma^{(2)}_{A^2}]^{\mu\nu}_{ab}(p) = Z_A(p) p^2 \Pi^{\mu\nu}_T(p)\delta_{ab} \qquad [\Gamma^{(2)}_{\bar{c}c}]_{ab}(p) = Z_c(p) p^2 \delta_{ab}$$





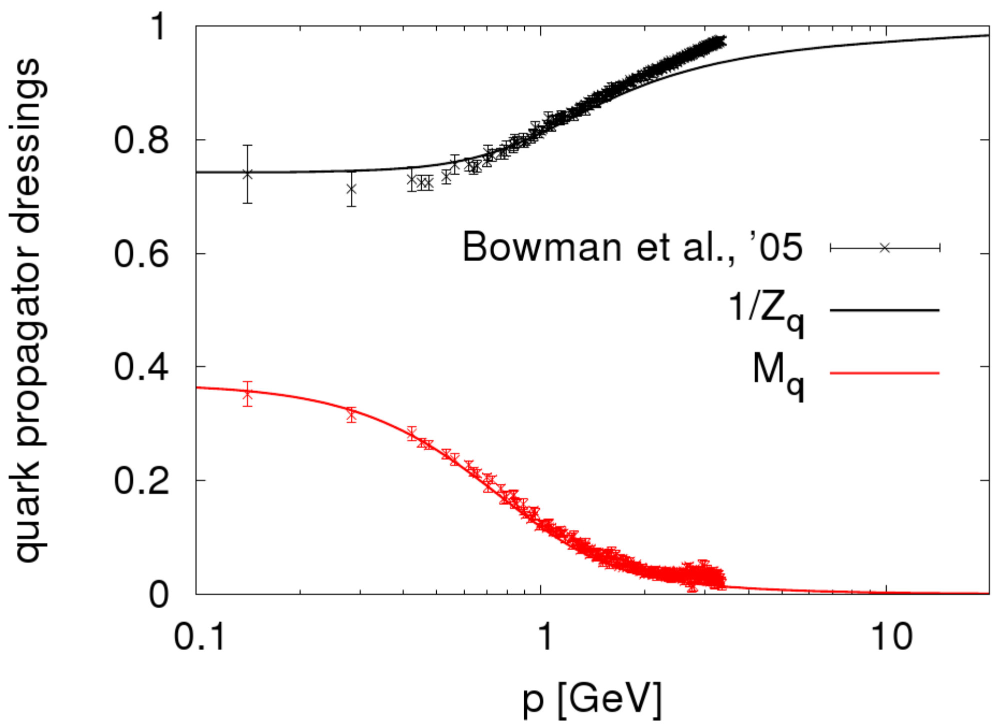
- Confinement from correlation functions:
  - Fister, Pawlowski, Phys.Rev. **D88** (2013) 045010
  - Braun, Gies, Pawlowski, Phys.Lett. **B684** (2010) 262-267

- Finite T results in preparation
  - Cyrol, Mitter, Pawlowski, NSt in prep

Quenched Quark Propagator

From the full matter system using quenched gluon propagator as only input

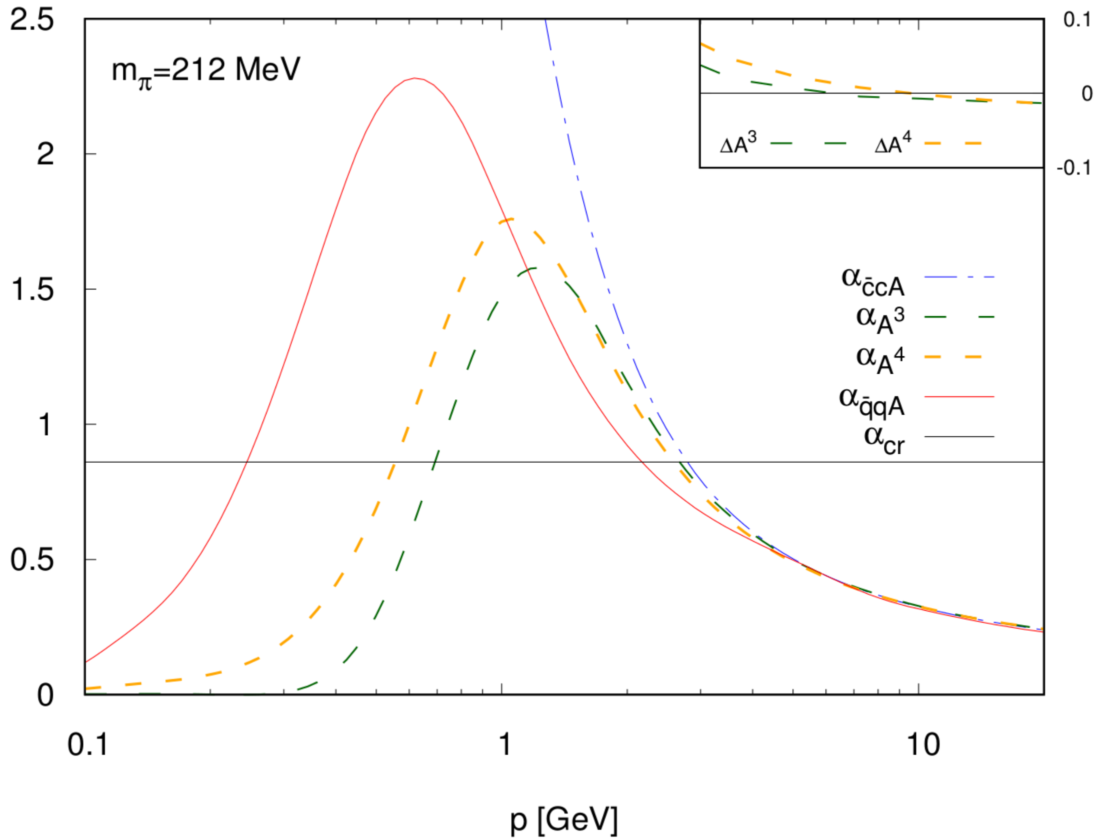
$$\Gamma^{(2)}_{\bar{q}q}(p) = Z_q(p)(\mathrm{i}\not{p} + M_q(p))$$



Very good agreement with (quenched) lattice results!

- Mitter, Pawlowski, NSt Phys.Rev. **D91** (2015) 054035

Running Couplings & STI



$$\alpha_{\bar{c}cA}(\bar{p}) = \frac{1}{4\pi}\frac{\left(\lambda^{(1)}_{\bar{c}cA}(\bar{p})\right)^2}{Z_A(\bar{p})\, Z_c^2(\bar{p})},$$

$$\alpha_{A^3}(\bar{p}) = \frac{1}{4\pi}\frac{\left(\lambda^{(1)}_{A^3}(\bar{p})\right)^2}{Z_A^3(\bar{p})},$$

$$\alpha_{A^4}(\bar{p}) = \frac{1}{4\pi}\frac{\lambda^{(1)}_{A^4}(\bar{p})}{Z_A^2(\bar{p})}.$$

$$\alpha_{\bar{q}qA}(\bar{p}) = \frac{1}{4\pi}\frac{\left(\lambda^{(1)}_{\bar{q}qA}(\bar{p})\right)^2}{Z_A(\bar{p})\, Z_q^2(\bar{p})}.$$

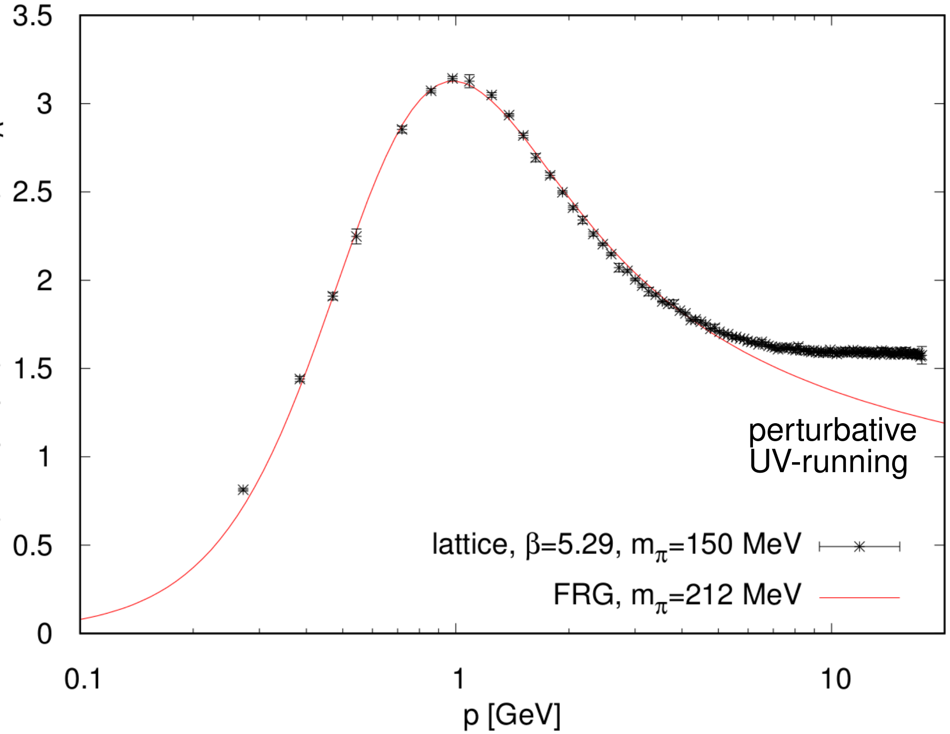
- Chiral symmetry breaking **very sensitive to correct semi-pert. running**
- Present solution: constrain classical tensor structure by **Slavnov-Taylor identity** in the semi-perturbative regime $\bar{p} \geq \Lambda_{\mathrm{STI}}$ ➢ Davydychev, P. Osland, and L. Saks (2001)

$$\lambda^{(9)}_{\bar{q}qA}(\bar{p}) = \frac{Z_q(\bar{p})}{Z_c(\bar{p})}\left[\lambda^{(1)}_{cqQ_q}(\bar{p}) - \frac{3}{2}\,\bar{p}^2\,\lambda^{(4)}_{cqQ_q}(\bar{p})\right]$$

- long. proj. class. quark-gluon vertex
- quark-ghost scattering kernel

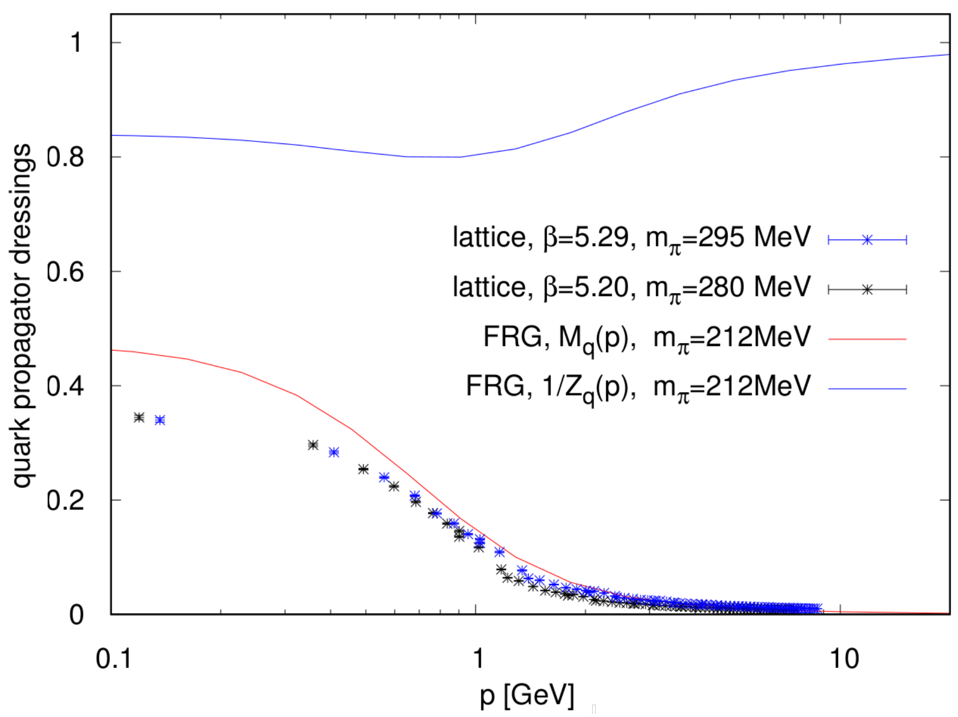
Unquenched Propagators

$$[\Gamma^{(2)}_{A^2}]^{\mu\nu}_{ab}(p) = Z_A(p) p^2 \Pi^{\mu\nu}_T(p)\delta_{ab}$$



- Cyrol, Mitter, Pawlowski, NS in prep
- Lattice: Sternbeck et al
  PoS LATTICE2012, 243 (2012)

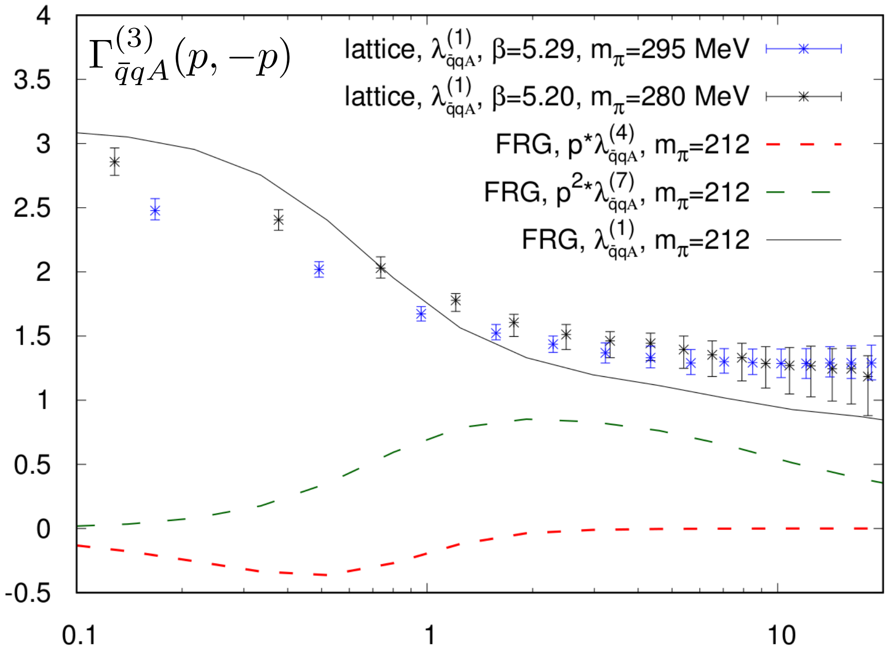
$$\Gamma^{(2)}_{\bar{q}q}(p) = Z_q(p)(\mathrm{i}\not{p} + M_q(p))$$



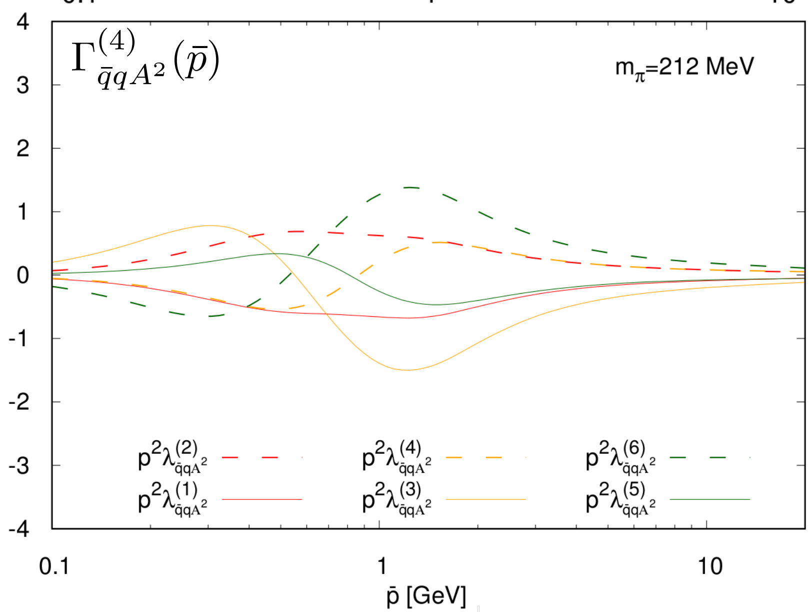
- Cyrol, Mitter, Pawlowski, NS in prep
- Lattice: Oliveira et al 1605.09632

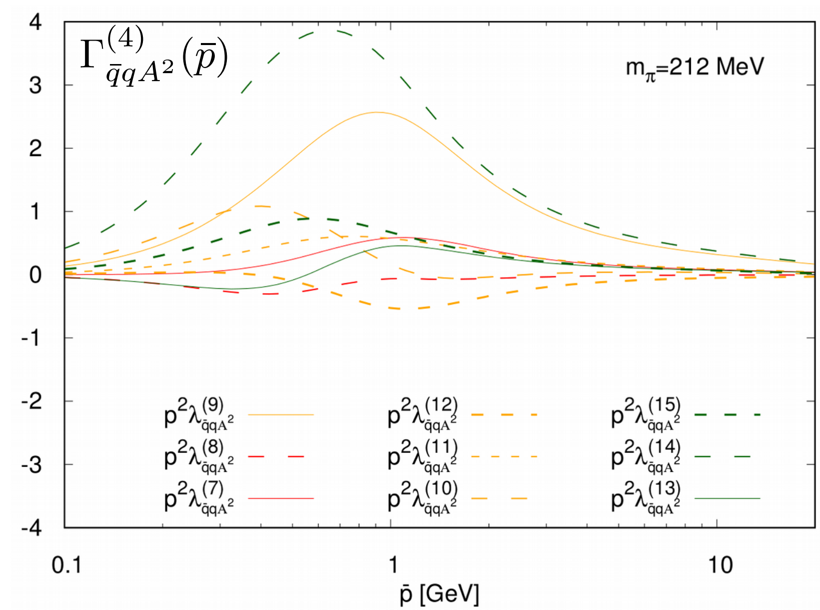
Still to be done: proper scale-matching to the lattice (curvature vs. pole mass)

Quark-Gluon Interactions



- Three quantitatively important tensor structures
  ch. sym.: $\mathcal{T}^{(1)}_{\bar{q}qA} = -\mathrm{i}\gamma^\mu$
  $\mathcal{T}^{(7)}_{\bar{q}qA} = -\frac{\mathrm{i}}{2}[\not{p},\not{q}]\gamma^\mu$
  ch. sym. br.: $\mathcal{T}^{(4)}_{\bar{q}qA} = (\not{p}+\not{q})\gamma^\mu$
- **Full (3d) momentum resolution** required for quantitative accuracy
- First direct calculation of the 2-quark-2-gluon vertex





Summary

FQCD, a quantitative continuum approach to QCD

- ✔ Quenched QCD, YM Theory,...
- ❑ Unquenched QCD as a prerequisite for finite T and mu

Unquenched QCD

- ❑ First self-consistent solution of the full system
- ❑ Correct semi-perturbative running crucial for quantitative accuracy

Stay tuned...

- ❑ Finite temperature (YM)
- ❑ Directly calculated spectral functions
  - ➢ NSt 1611.05036
  - ➢ Pawlowski, NSt Phys.Rev. **D92** (2015) 9, 094009
  - ➢ Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010...
- ❑ Transport coefficients
  - ➢ Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002
- ❑ Bound state properties
- ❑ ...

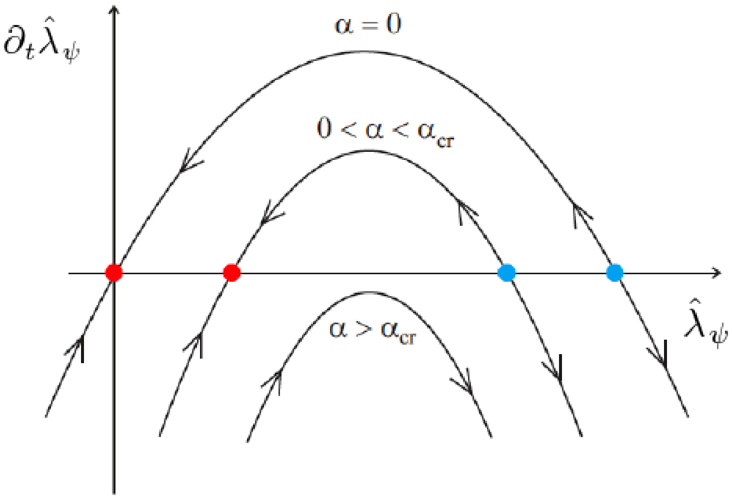
Thank you for your attention!

Backup

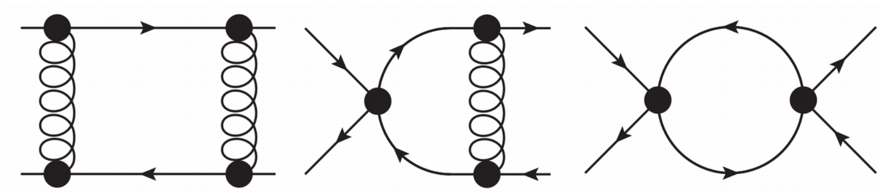
Chiral symmetry breaking

χSB <-> resonance in 4-quark interaction (pion pole)

β-function:



$$k\partial_k \hat{\lambda}_\psi = (d-2)\hat{\lambda}_\psi - a\hat{\lambda}_\psi^2 - b\hat{\lambda}_\psi g^2 - cg^4$$



- Review: Braun J.Phys. **G39** (2012) 033001

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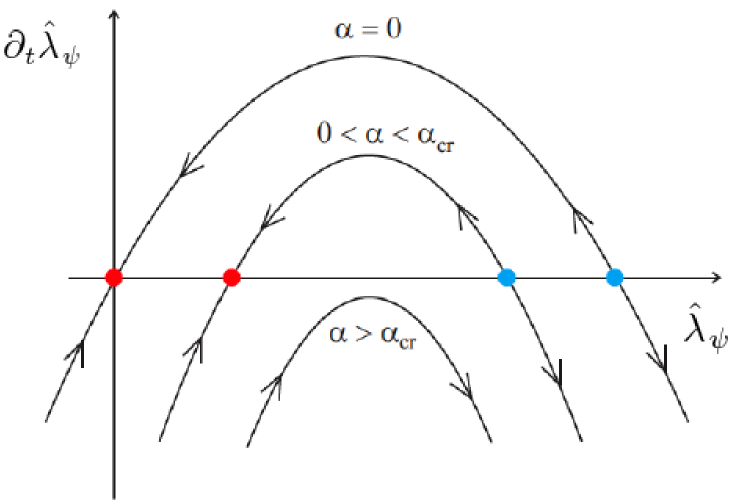

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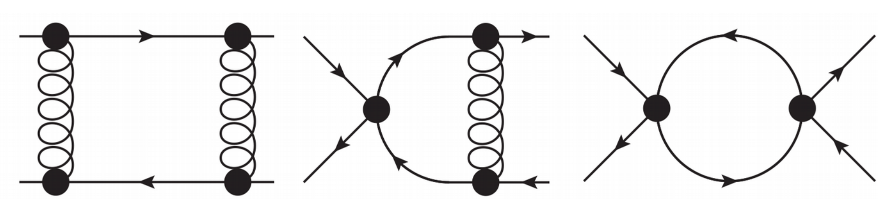

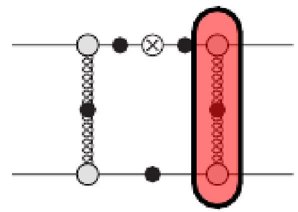

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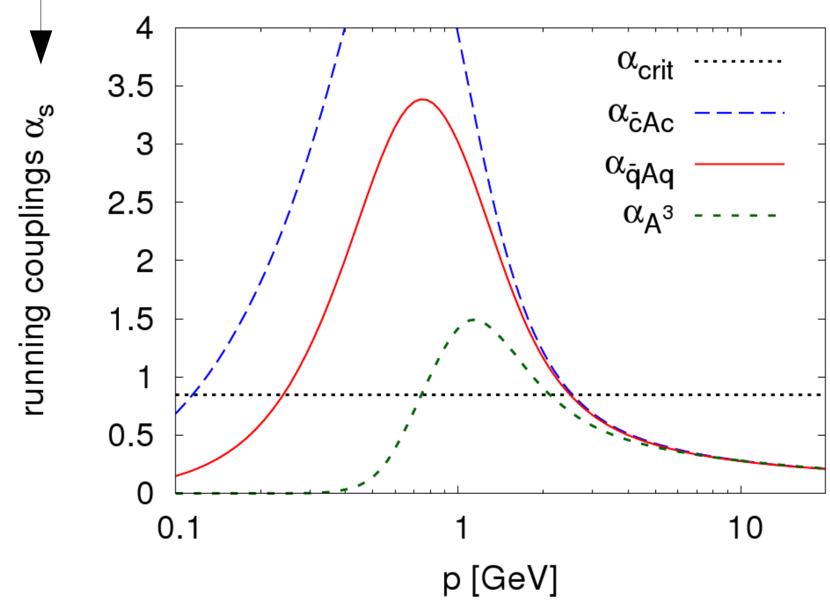
$$\alpha_{\bar{q}Aq}(p) = \frac{Z^2_{\bar{q}Aq}(\bar{p})}{4\pi Z_A(p) Z_q^2(p)}$$

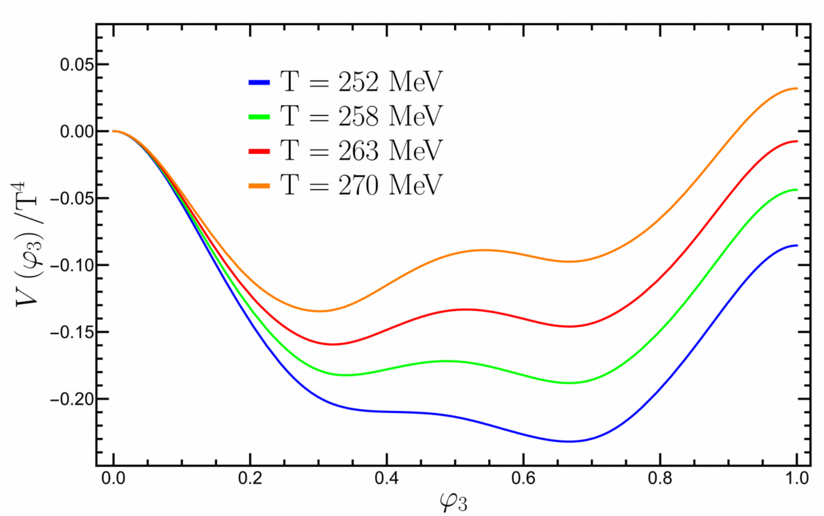
$$\alpha_{A^3}(p) = \frac{Z^2_{A^3}(\bar{p})}{4\pi Z_A^3(p)}$$



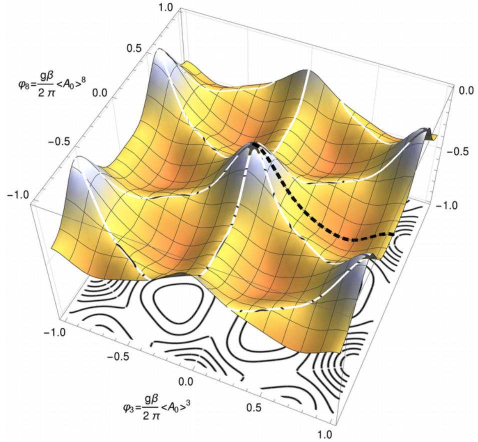
- Review: Braun J.Phys. **G39** (2012) 033001

Confinement

$V(\langle A_0 \rangle)$ from YM propagators



$L(\langle A_0 \rangle)$



Confined:

$$\bar{\varphi}_3 = \frac{2}{3}$$

$$L(\bar{\varphi}_3, 0) = 0$$

- Pawlowski, Scherzer, Strodthoff, Wink in prep.
- Herbst, Luecker, Pawlowski, (2015), 1510.03830
- Fister, Pawlowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawlowski, Phys.Lett. B684 (2010)

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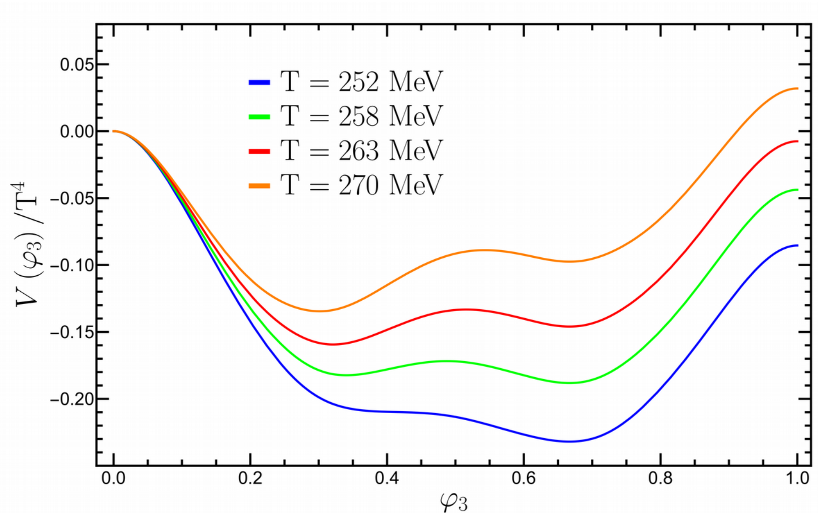
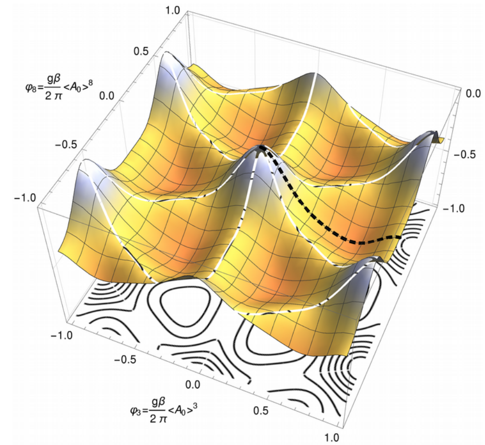


$L(\langle A_0 \rangle)$



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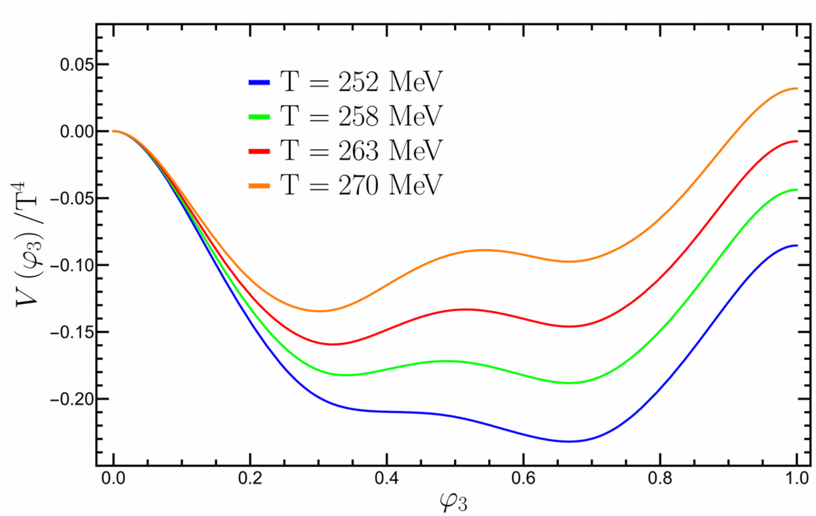

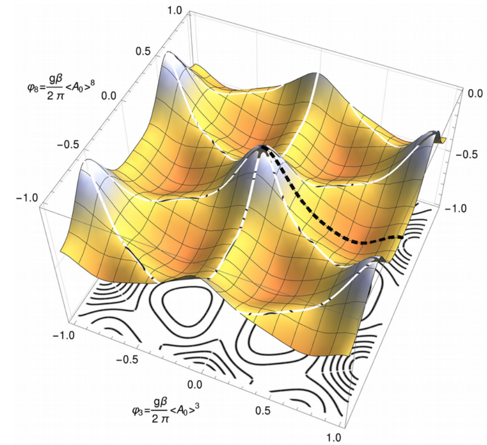

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$$L(\langle A_0 \rangle)$$

- Pawlowski, Scherzer, Strodthoff, Wink in prep.
- Herbst, Luecker, Pawlowski, (2015), 1510.03830
- Fister, Pawlowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawlowski, Phys.Lett. B684 (2010)

Order parameters:

$\bar{\varphi}_3$, $L(\langle A_0 \rangle)$: most easily computed in functional methods

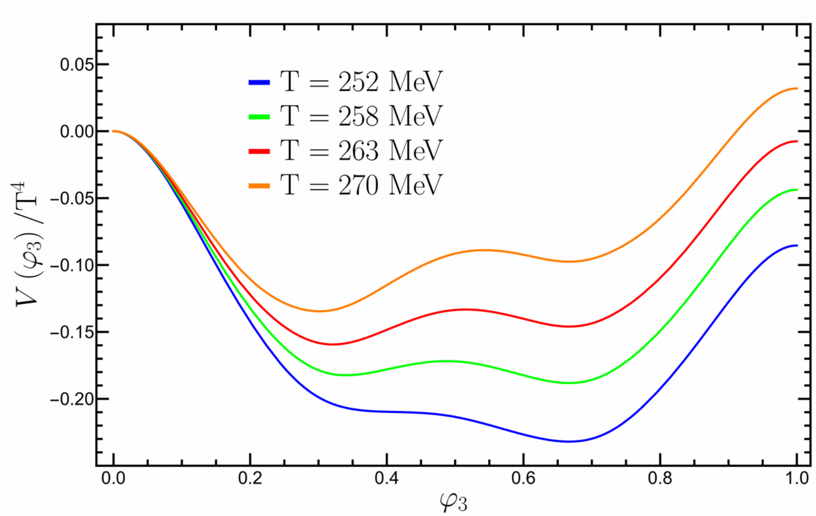
$\langle L(A_0) \rangle$: computed on the lattice; now also in the FRG

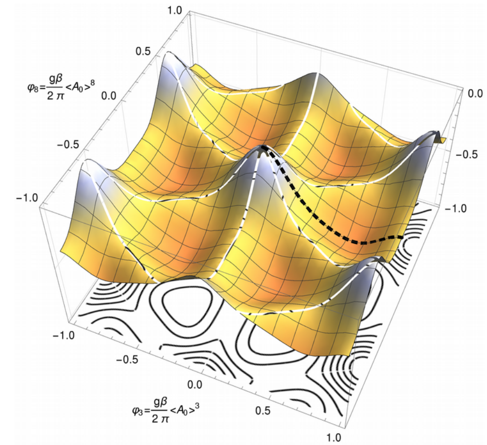
$$\langle L(A_0) \rangle \leq L(\langle A_0 \rangle)$$

$$\langle L(A_0) \rangle = 0 \iff L(\langle A_0 \rangle) = 0$$

Confinement

$V(\langle A_0 \rangle)$ from YM propagators





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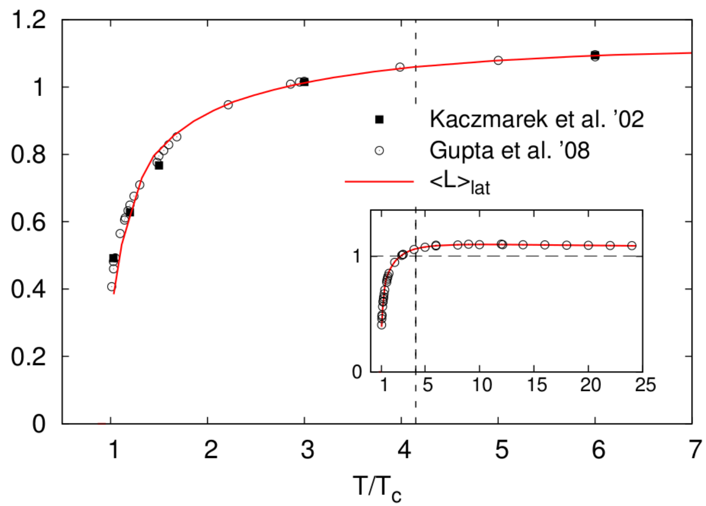
- Fister, Pawlowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawlowski, Phys.Lett. B684 (2010)

$\langle L(A_0) \rangle$ from $L(\langle A_0 \rangle)$



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$\langle L(A_0) \rangle$ computed on the lattice; now also in the FRG

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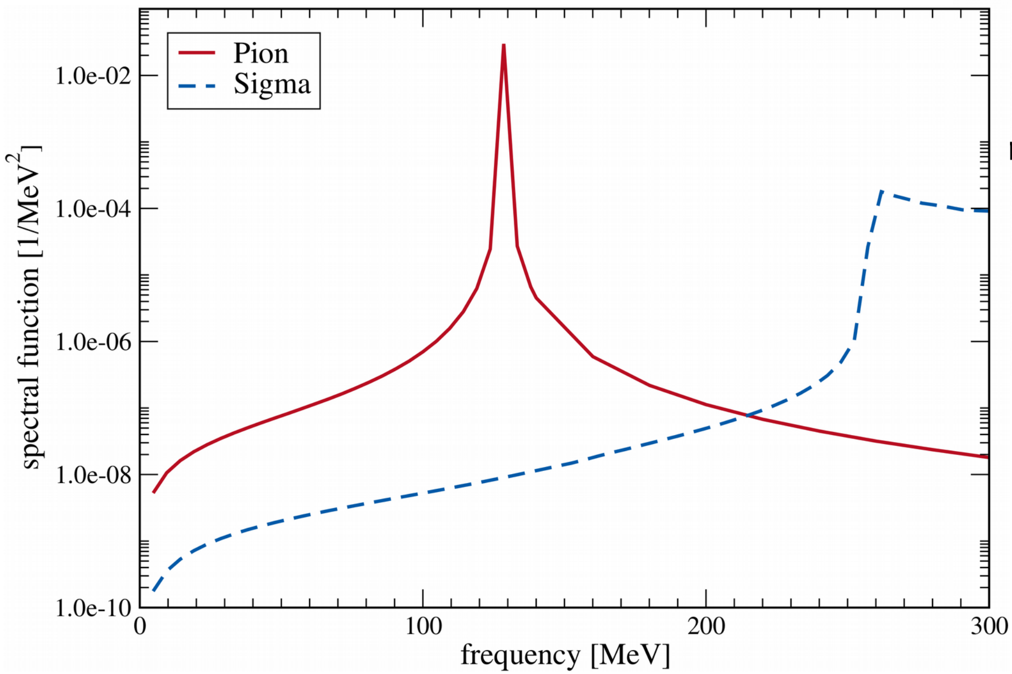
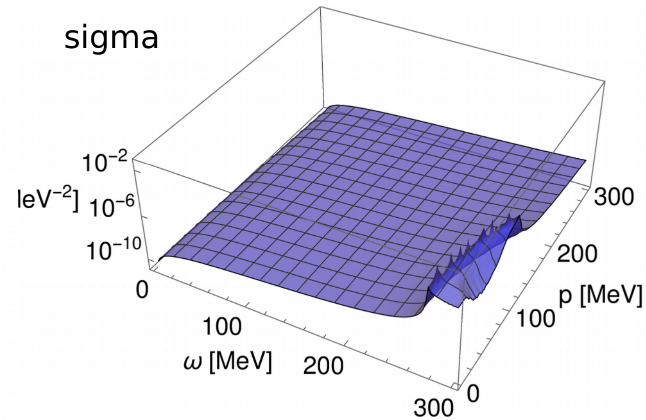
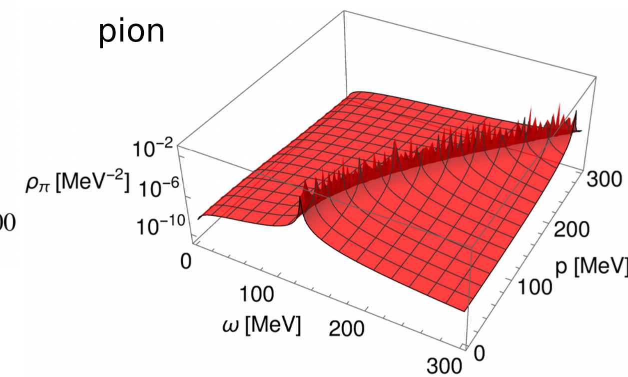
- Herbst, Luecker, Pawlowski, (2015), 1510.03830

Spectral Functions

O(N) at T=0, full momentum dependence







- NSt, in prep

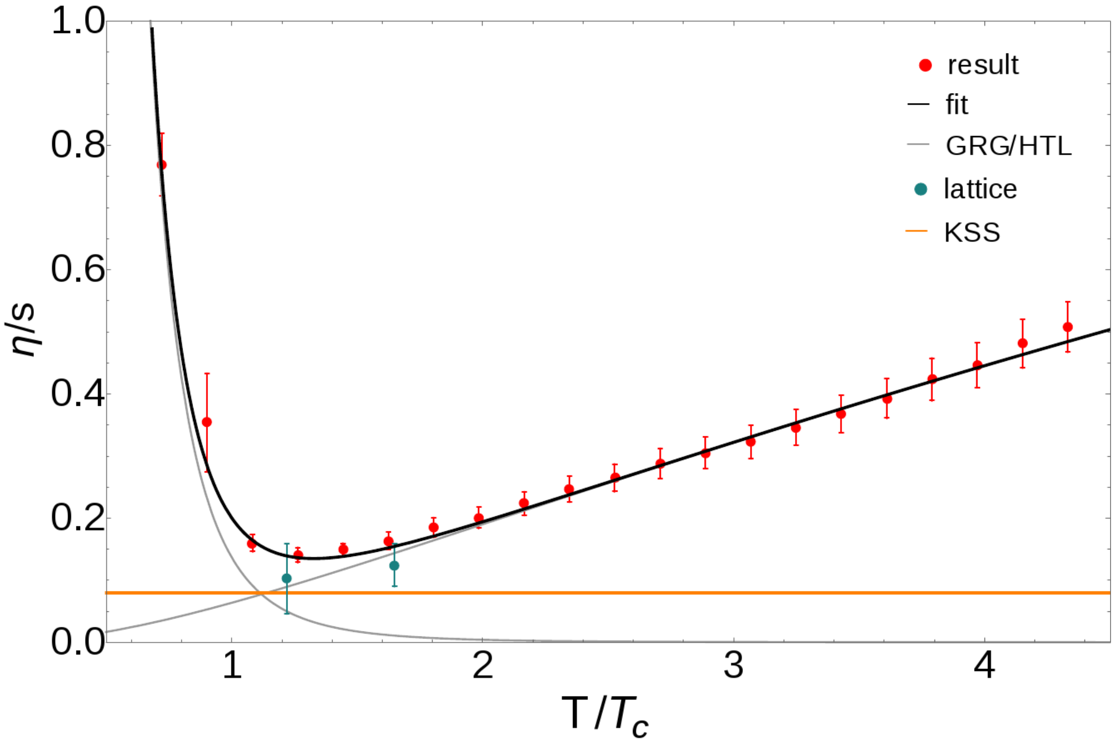
Summary

- ✔ Directly calculated spectral functions
- ✔ Tested in scalar and Yukawa models at T,μ>0
- ✔ Allows the inclusion of full momentum dependence

- ❑ Quark & gluon spectral functions in full QCD

- NSt 1611.05036
- Pawlowski, NSt
  Phys.Rev. **D92** (2015) 9, 094009
- Tripolt, NSt, von Smekal, Wambach
  Phys.Rev. **D89** (2014) 034010
- Kamikado, NSt, von Smekal, Wambach
  Eur.Phys.J. **C74** (2014) 2806

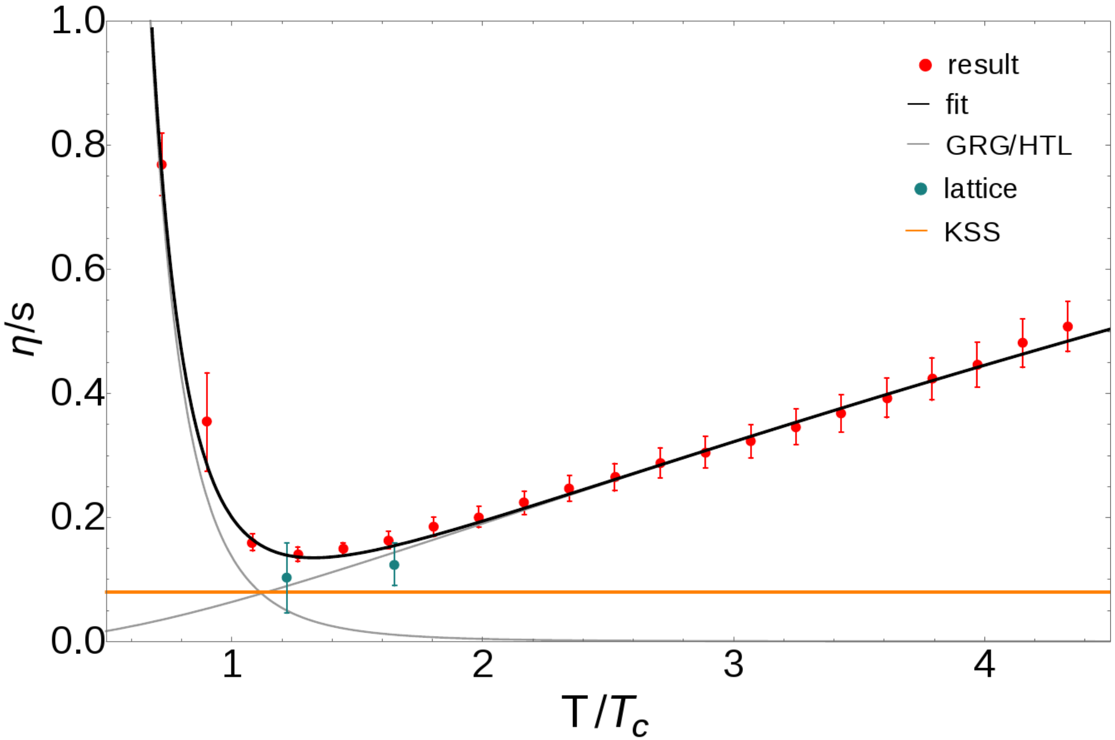
η/s in Yang-Mills Theory



T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

η/s in Yang-Mills Theory



T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

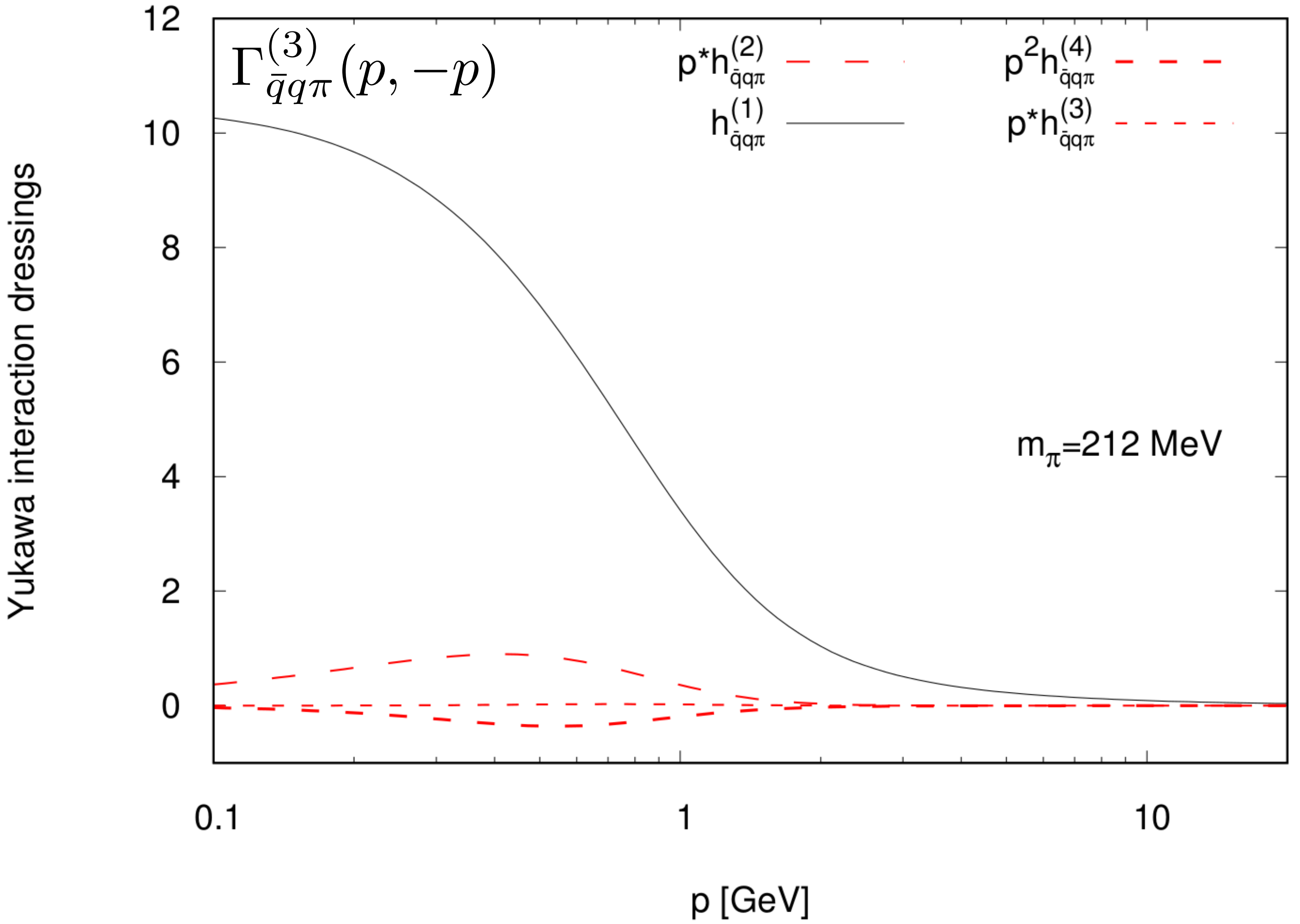
Direct sum: $$\frac{\eta}{s}(T) = \frac{a}{\alpha_s(cT/T_c)^{\gamma}} + \frac{b}{(T/T_c)^{\delta}}$$

$$\gamma = 1.6 \quad a = 0.15 \quad b = 0.14 \quad c = 0.66 \quad \delta = 5.1$$

High T: **consistent with HTL-resummed pert. theory** (fixing γ) supporting quasiparticle picture

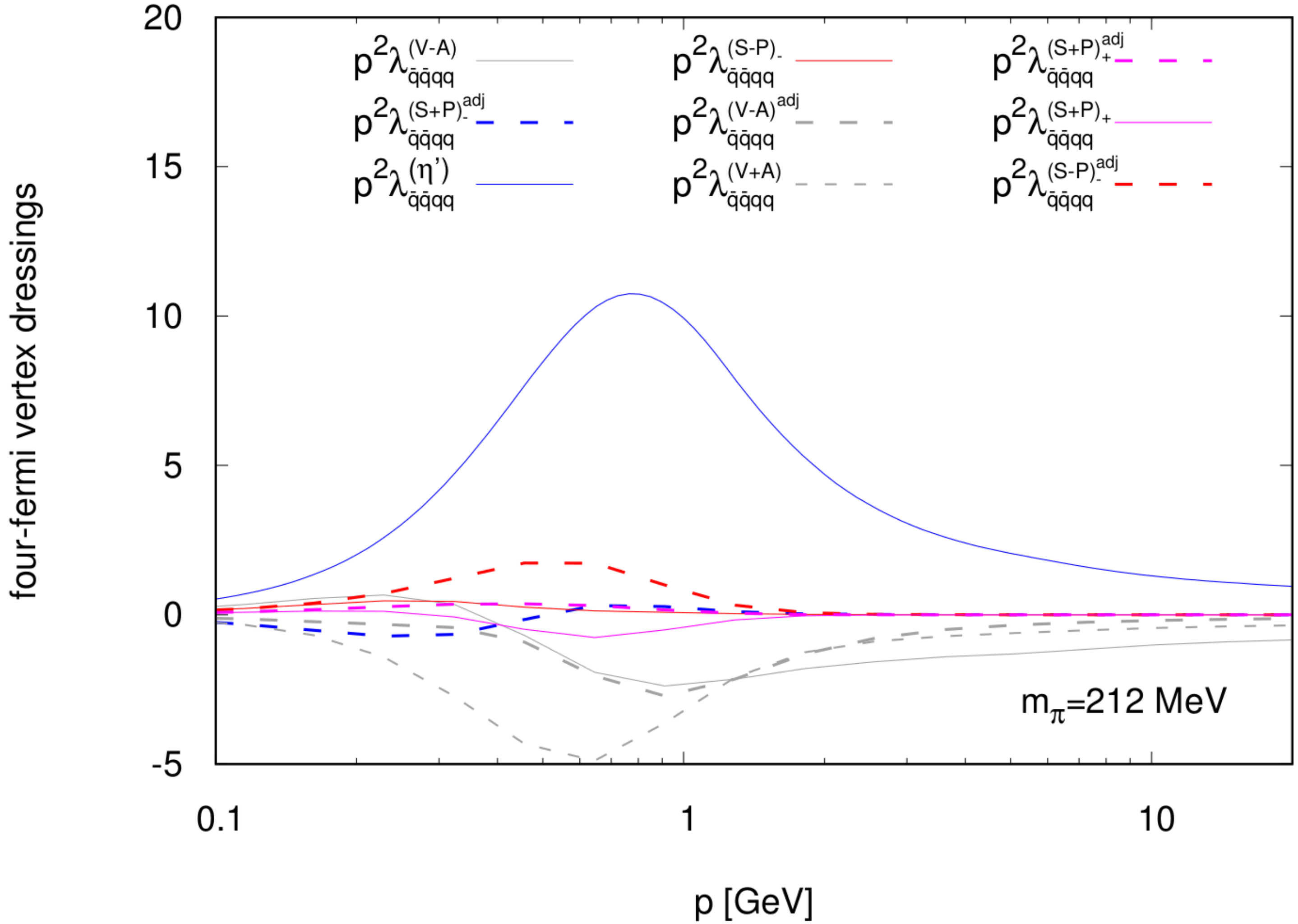
Small T: algebraic decay **glueball resonance gas**

Quark-Meson Interactions



4-Fermi Interactions

$\Gamma^{(4)}_{\bar{q}\bar{q}qq}(p,p,-p)$



Ghost Propagator

$$\Gamma^{(2)}_{\bar{c}c}(p) = Z_c(p)p^2$$